Math 106330 Algebraic Geometry: Schemes, Sheaves, and Cohomology
3 Credit Hours, Winter 2023-2024

Instructor: Howard Nuer
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Class Meeting: Mon 9:30-11:30 in Amado 719, Wed 14:30-15:30 in Amado 719.

COURSE DESCRIPTION/COURSE OBJECTIVES: Welcome to Math 106330! I will give a comprehensive treatment on schemes, sheaves, cohomology. In order to cover all of this ground, I will sometimes leave the verification of simpler facts to the reading or the homework. In addition, there are many fundamental results in algebraic geometry whose proof is less relevant than their applications. In these cases, I will focus classtime on the meaning of the theorem and using it, while leaving the proof to the reading.

A student completing the course in good standing will be able to produce examples exhibiting various algebro-geometric phenomena and compute the cohomology of sheaves arrising in standard examples. Such a student will be in the perfect position to continue on to further study in algebraic geometry, arithmetic geometry and number theory, geometric representation theory, and symplectic geometry.

WEBPAGE: Moodle Course Page

PREREQUISITES: Modern Algebra 1 (106380). I will have ready by the first day of class a short summary of the results from that course that we will use in this course. Although not a prerequisite, any prior exposure to differentiable manifolds will help provide context to this course.

TEXTBOOK: Hartshorne, *Algebraic Geometry*. This is the standard textbook for graduate algebraic geometry. It's quick (relatively) and teaches the language of schemes and cohomology, but the actual geometry seems to be missing until you know how everything relates. I'll try to provide that insight in class as we go along.

ADDITIONAL REFERENCES: The following is a list of references that I think will provide you with a different perspective or discuss more examples.

- Ravi Vakil, *The Rising Sea: Foundations of Algebraic Geometry*. These are a great and free set of notes from one of the modern masters who has been giving a course based on them for many years running. He fleshes out a lot of the material in Hartshorne, but the current version of the notes is 783 pages…

- Ulrich Görtz and Torsten Wedhorn, *Algebraic Geometry 1: Schemes with Examples and Exercises*. This long tome is like the child of Ravi's notes and Hartshorne. It's very general, but also very detailed and emphasizes a lot of the same graded-ring aspects that Ravi does.
- Q. Liu, *Algebraic Geometry and Arithmetic Curves*. This is a little less harsh than Hartshorne and is great for emphasizing the arithmetic side of things.
- J. Harris, *Algebraic Geometry: A First Course*. This book is great for examples and seeing the classical geometry involved in all of the abstract language.
- D. Eisenbud and J. Harris, *The Geometry of Schemes*. This book is very readable and emphasizes the geometry behind the seemingly abstract language of schemes.
- P. Griffiths and J. Harris, *Principles of Algebraic Geometry*. This book presents the subject from the complex geometry perspective, emphasizing complex functions and complex analysis. A lot of beautiful classical algebraic geometry is discussed.
- *The Stacks Project*. This is the modern reincarnation of EGA and SGA in English, online, and fully searchable. It is the encyclopedia of algebraic geometry.
- D. Eisenbud, *Commutative Algebra with a View Toward Algebraic Geometry*. This book contains all of the commutative algebra we will need (and more) and explains what the algebra means geometrically.

GRADING: The final grade will be based on Homework (25%) and a three hour final exam (75%).

HOMEWORK: I have found that students won't do the homework if it's just recommended and is not collected and checked regularly. It is especially important to invest time and energy into the homework for this course as this material is particularly dense, and it takes time to become fluent in its language. The only way to understand it is to grapple with it at close range by working problems. Moreover, in class I will focus on the techniques, methods. and examples of algebraic geometry, leaving certain technical details to the reading or the homework. I strongly recommend you get together in groups and work through the homework problems.

There will be approximately 10-13 homework assignments. In calculating the homework component of the grade, I will drop the lowest two homework grades. Each homework assignment will consist of 3-4 Exercises and 7-9 Problems. The Exercises will be taken from points arrising in class that I want to leave you to fill in. The Problems will be exercises from Hartshorne, the course textbook.

As there is not a grader for this course, I will grade one exercise and one problem from each hoemwork assignment to determine the grade for the assignment (without letting you know which ones of course). I will post solutions to the homework assignments at the end of the semester before the exam.

READING: Although I have no way of or interest in grading you directly on this, there will be assigned reading from the textbook. This will be any reference I give for the proof of something.

Yes I really intend you to look at the proof in the textbook, and I will ask you about this on the exam. In fact, …

FINAL EXAM: 3 hour exam. Moed A 18/02/24, Moed B 18/03/24. To further encourage you to invest in the homework and reading, the problems on final exam will be take entirely from the homework and the assigned reading.

LIST OF ANTICIPATED TOPICS AND TEXTBOOK PROBLEMS (OFFICIAL ASSIGNMENTS MAY VARY SLIGHTLY):

Topic	Section in Textbook	Assigned Problems	At least read these problems
Sheaves	II.1	1.2,1.3,1.4,1.6,1.8,1.15,1.16,1.21,1.22	1.9, 1.10, 1.11, 1.12, 1.14, 1.17, 1.18, 1.19
Schemes	II.2	2.3, 2.4, 2.8, 2.12, 2.14, 2.16, 2.17, 2.18, 2.19	2.13
First Properties of Schemes	II.3	3.4, 3.5, 3.7, 3.11, 3.14, 3.15, 3.18, 3.20, 3.21	3.1, 3.2, 3.3, 3.9, 3.12, 3.13
Separated and Proper Morphisms	II.4	4.1, 4.2, 4.3, 4.4, 4.5, 4.6, 4.7, 4.9, 4.12	4.8, 4.10, 4.11
Sheaves of Modules	II.5	5.1, 5.2, 5.3, 5.6, 5.7, 5.8, 5.9, 5.11, 5.13, 5.16, 5.18	5.4, 5.5, 5.10, 5.14, 5.15, 5.17
Divisors	II.6	6.1, 6.2, 6.3, 6.4, 6.5, 6.6, 6.8, 6.9, 6.12	6.7, 6.10, 6.11
Projective Morphisms	II.7	7.2, 7.3, 7.4, 7.5, 7.6, 7.7, 7.8, 7.11, 7.12	7.10, 7.13, 7.14
Differentials	II.8	8.1, 8.2, 8.3, 8.4, 8.5, 8.6, 8.7, 8.8	
Derived Functors and Cohomology of Sheaves	III.1 and III.2	2.1, 2.2, 2.6, 2.7	2.3, 2.4
Cohomology of a Noetherian Affine Scheme	III.3	3.1, 3.2, 3.5, 3.6, 3.7, 3.8	3.3, 3.4
Čech Cohomology	III.4	4.1, 4.2, 4.3, 4.4, 4.5, 4.6, 4.7, 4.9, 4.11	4.8, 4.10
Cohomology of Invertible Sheaves on Projective Space	III.5	5.1, 5.2, 5.3, 5.5, 5.6, 5.7, 5.8	5.4, 5.9

Topic	Section in Textbook	Assigned Problems	At least read these problems
Ext Groups and Sheaves	III.6	6.1, 6.2, 6.3, 6.4, 6.5, 6.6, 6.7, 6.8	6.9, 6.10
Serre Duality	III.7	7.1, 7.2, 7.3, 7.4	
Higher Direct Image of Sheaves	III.8	8.1, 8.2, 8.3, 8.4	
Flat Morphisms	III.9	9.1, 9.2, 9.3, 9.4, 9.6, 9.7, 9.9, 9.10, 9.11	9.5
Zariski's Main Theorem, Semicontinuity, and the Cohomology and Base Change Theorem	III.11 and III.12	11.1, 11.2, 11.4, 11.8, 12.1, 12.2, 12.3, 12.4, 12.5, 12.6	11.3, 11.5, 11.6, 11.7

ATTENDANCE POLICY: Registered students are expected to attend and participate in all of the lectures and should notify me before any planned absence. That being said, attendance won't be recorded.

ELECTRONIC COMMUNICATION: The best ways to reach me are to catch me before & after class, in my office hours, and by email. I will generally try to reply to emails within 1-2 days; please feel free to resend again after that. For questions involving serious mathematical content, it is generally better to ask in class since others will likely have the same question.