Math 106803 Topics in Geometry: Complex Geometry
3 Credit Hours, Spring 2023-2024

Instructor: Howard Nuer
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Class Meeting: Sun 13:30-15:30 and Wed 10:30-11:30 in Amado 619.

COURSE DESCRIPTION/COURSE OBJECTIVES: Welcome to Math 106803! This course will study complex manifolds, differentiable manifolds with an atlas whose transition functions are holomorphic. These manifolds arise naturally when studying zeros of polynomials and are ubiquitous throughout physics.

As one sees in a course on complex analysis, the transition from smooth functions to holomorphic ones has numerous consequences and imposes rigid restrictions on the geometry of complex manifolds, often giving the subject a very algebraic flavour. This provides new information about familiar invariants of manifolds: besides usual de Rham cohomology of smooth manifolds, complex manifolds have a cohomology theory associated to their complex structure: Dolbeault cohomology. In the most interesting cases, that of Kähler manifolds, there is in fact a close relation between these two cohomology theories embodied in the Hodge decomposition.
On the other hand, in higher dimension, another manifestation of this rigidity is the failure of Whitney's embedding theorem: no compact complex manifold of positive dimension can be embedded in C^n. This raises the fundamental question: is there a natural space where compact complex manifolds live?

We will start with the foundations of almost complex structures and integrability, sheaf theory, sheaf cohomology, Hermitian and holomorphic vector bundles and line bundles in particular. We then move on to study elliptic operator theory and its application to the Hodge decomposition of cohomology on Kähler manifolds and proceed to proving Kodaira's embedding theorem, which characterises which Kähler manifolds can be embedded in CP^n, giving a partial answer to the fundamental question above.

A student completing the course in good standing will be able to understand the myriad of applications of complex geometry throughout mathematics and physics. Most importantly, the student will have the requisite background to study complex algebraic geometry, mirror symmetry, and symplectic geometry, in addition to going further in the geometry of complex manifolds.

WEBPAGE: Moodle Course Page

PREREQUISITES: Complex Functions (104122), Differentiable Manifolds (106723), Algebraic Topology (106383). I will do a quick review of some relevants results from single-variable compelx analysis with any proofs left to the relevant results of the textbook where detailed proofs can be found. In case your course on differentiable manifolds did not cover it, the textbook has a nice appendix (Appendix A) on the classical de Rham approach to understanding singular cohomology in terms of smooth differential forms. Any prior exposure to these ideas will help motivate what we will study when it comes to holomorphic differential forms.

TEXTBOOK: Daniel Huybrechts, *Complex Geometry: An Introduction*. This is becoming the standard textbook for geometry students to approach the differential geometry of complex manifolds. It's quick (relatively) and teaches very precisely and carefuly the subtle passage from the real world to the complex world, especially when it comes to differential forms. At the same time, the book covers a lot of examples and covers more than the standard first course, with lots of important hot topics relegated to appendices to invdividual chapters. We hope to cover the material from chapters 1-5. I will leave some of the more involved proofs for reading in the textbook and similarly with very technical lemmas from the linear algebra section, which are best done from the textbook. For any big results we cover that even the textbook doesn't prove, the textbook contains nice references where proofs may be found.

ADDITIONAL REFERENCES: The following is a list of references that I think will provide you with a different perspective or discuss more examples.

- P. Griffiths and J. Harris, *Principles of Algebraic Geometry*.
- Voisin, *Complex Algebraic Geometry I*
- R.O. Wells, *Differential Analysis on Complex Manifolds*
- Barth, Peters, and Van de Ven, *Compact Complex Surfaces*

GRADING: The final grade will be based on Homework (25%) and a three hour final exam (75%).

HOMEWORK: I have found that students won't do the homework if it's just recommended and is not collected and checked regularly. It is especially important to invest time and energy into the homework for this course. The only way to understand it is to grapple with it at close range by working problems. Moreover, as mentioned above, in class I will focus on the important techniques, methods, and examples of complex geometry, leaving certain technical details to the reading. Some of it, I will leave for the homework. I strongly recommend you get together in groups and work through the homework problems.

There will be approximately 10 homework assignments. In calculating the homework component of the grade, I will drop the lowest two homework grades. Each homework assignment will consist of 4-6 Problems. The problems will be exercises I write or take straight from the course textbook.

As there is not a grader for this course, I will grade two problems from each homework assignment to determine the grade for the assignment (without letting you know which ones of course).

READING: Although I have no way of or interest in grading you directly on this, there will be implied assigned reading from the textbook. This will be any reference I give for the proof of something from the textbook. If you are taking this course for a grade, I really intend you to look at the proof in the textbook, and I may ask you about this on the exam.

FINAL EXAM: 3 hour exam. Moed A 28/08/24, Moed B 23/09/24. To further encourage you to invest in the homework and reading, the problems on the final exam will be taken entirely from lectures, the homework, and the assigned reading.

ATTENDANCE POLICY: Registered students are expected to attend and participate in all of the lectures and should notify me before any planned absence. That being said, attendance won't be recorded.

ELECTRONIC COMMUNICATION: The best ways to reach me are to catch me before & after class, in my office hours, and by email. I will generally try to reply to emails within 1-2 days; please feel free to resend again after that. For questions involving serious mathematical content, it is generally better to ask in class since others will likely have the same question.